CHAPTER 3

ONE-DIMENSIONAL LINEAR AND PERIODIC HEAT FLOW

3-1. Thermal regime.

The seasonal depths of frost and thaw penetration in soils depends upon the thermal properties of the soil mass, the surface temperature (upper boundary condition) and the thermal regime of the soil at the start of the freezing or thawing season. Many methods are available to estimate frost and thaw penetration depths and surface temperatures. Some of these are summarized in appendix B. This chapter concentrates on some techniques that require only relatively simple hand calculations. For the computational methods discussed below, the initial ground temperature is assumed to uniformly equal the mean annual air temperature of the particular site under consideration. The upper boundary condition is represented by the surface freezing (or thawing) index.

3-2. Modified Berggren equation.

a. The depth to which 32°F temperatures will penetrate into the soil mass is based upon the "modified" Berggren equation, expressed as:

$$X = \lambda \sqrt{\frac{48\,K\,nF}{L}}$$

or (eq 3-1)

$$X = \lambda \sqrt{\frac{48\,K\,nI}{L}}$$

where

X= depth of freeze or thaw (ft)

K= thermal conductivity of soil (Btu/ft hr °F)

L= volumetric latent heat of fusion (Btu/ft^3)

n= conversion factor from air index to surface index (dimensionless)

F= air freezing index (°F-days)

I = air thawing index (°F-days)

λ = coefficient that considers the effect of temperature changes in the soil mass (dimensionless).

The λ coefficient is a function of the freezing (or thawing) index, the mean annual temperature of the site, and the thermal properties of the soil. Freeze and thaw of low-moisture-content soils in the lower latitudes is greatly influenced by this coefficient. It is determined by two factors: the thermal ratio α and the fusion parameter μ. These have been defined in paragraph 2-1. Figure 3-1 shows λ as a function of α and μ.

b. A complete development of this equation and a discussion of the necessary assumptions and simplifications made during its development are not presented here. A few of the more important assumptions and some of the equation limitations are discussed below. The assumptions and limitations apply regardless of whether the equation is used to determine the depth of freeze or the depth of thaw.

(1) *Assumptions.* The mathematical model assumes one-dimensional heat flow with the entire soil mass at its mean annual temperature (MAT) prior to the start of the freezing season. It assumes that when the freezing season starts, the surface temperature changes suddenly (as a step function) from the mean annual temperature to a temperature v_s degrees below freezing and that it remains at this new temperature throughout the entire freezing season. Latent heat affects the model by acting as a heat sink at the moving frost line, and the model assumes that the soil freezes at a temperature of 32°F.

(2) *Limitations.* The modified Berggren equation is able to determine frost penetration in areas where the ground below a depth of several feet

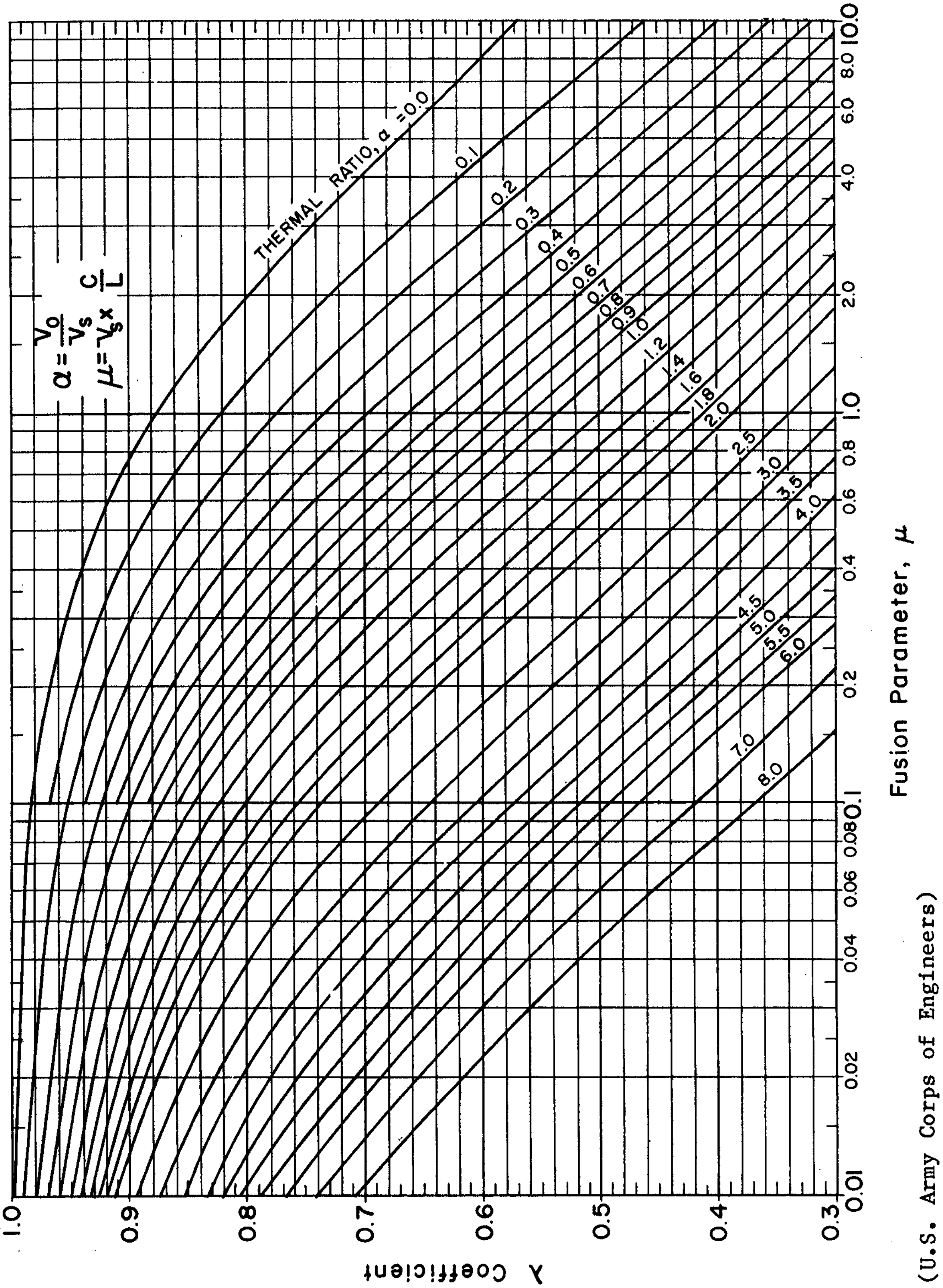


Figure 3-1. λ coefficient in the modified Berggren formula.

remains permanently thawed, or to determine thaw penetration in areas where the ground below a depth of several feet remains permanently frozen. These two conditions are similar in that the temperature gradients are of the same shape, although reversed with respect to the 32°F line. No simple analytical method exists to determine the depth of thaw in seasonal frost areas or the depth of freeze in permafrost areas, and such problems should be referred to HQDA (DAEN-ECE-G) or HQ AFESC. Numerical techniques and computer programs are available to solve more complex problems. Appendix B discusses some thermal computer models for computing freeze and thaw depths. The modified Berggren equation cannot be used successfully to calculate penetration over parts of the season. The modified Berggren equation does not account for any moisture movement that may occur within the soil. This limitation would tend to result in overestimated frost penetration (if frost heave is significant) or underestimated thaw penetration.

(3) *Applicability*. The modified Berggren equation is most often applicable in either of two ways: to calculate the multi-year depth of thaw in permafrost areas or to calculate the depth of seasonal frost penetration in seasonal frost areas. It is also sometimes used to calculate seasonal thaw penetration (active layer thickness) in permafrost areas.

3-3. Homogeneous soils.

The depth of freeze or thaw in one layer of homogeneous soil may be determined by means of the modified Berggren equation. A thin bituminous concrete pavement will not affect the homogeneity of this layer in calculations, but a portland-cement-concrete pavement greater than 6 inches thick should be treated as a multilayered system. In this *example* for homogeneous soils, determine the depth of frost penetration into a homogeneous sandy silt for the following conditions:

—Mean annual temperature (MAT) = 37.2°F.

—Surface freezing index (nF) = 2500 degree-days.

—Length of freezing season (t) = 160 days.

—Soil properties: γ_d = 100 lb/ft³, w = 15%.

The soil thermal properties are as follows:

—Volumetric latent heat of fusion,

$$L = 144(100)(0.15) = 2160 \text{ Btu/ft}^3. \qquad \text{(eq. 3-2)}.$$

—Average volumetric heat capacity,

$$C_{avg} = 100[0.17++(0.75\times0.15)]$$
$$= 28.3 \text{ Btu/ft}^3\ {}^\circ\text{F}. \qquad \text{(eq 3-3)}$$

—Average thermal conductivity,

K_f = 0.80 Btu/ft hr °F (fig. 2-3)

K_u = 0.72 Btu/ft hr °F (fig. 2-4)

$K_{avg} = 1/2\,(K_u + K_f)$ = 0.76 Btu/ft hr °F.

The λ coefficient is as follows:

—Average surface temperature differential,

$$v_s = nF/t = 2500/160$$
$$= 16.6^\circ\text{F (16.6°F below 32°F)}. \qquad \text{(eq 3-4)}$$

—Initial temperature differential,

$$v_o = MAT - 32 = 37.2 - 32.0 =$$
$$5.2^\circ\text{F (5.2° above 32°F)}. \qquad \text{(eq 3-5)}$$

—Thermal ratio,

$$\alpha = v_o/v_s = 5.2/16.6 = 0.33. \qquad \text{(eq 3-6)}$$

—Fusion parameter,

$$\mu = v_s\,(C/L) = 16.6(28.2/2160) = 0.20. \qquad \text{(eq 3-7)}$$

—Lambda coefficient,

$$\lambda = 0.89 \text{ (fig. 3-1)}. \qquad \text{(eq 3-8)}$$

Estimated depth of frost penetration,

$$X = \lambda\sqrt{\frac{48\,K\,nF}{L}} = 0.89\sqrt{\frac{48(0.76)(2500)}{2160}} = 5.8 \text{ ft.} \qquad \text{(eq 3-9)}$$

3-4. Multilayer soils.

A multilayer solution to the modified Berggren equation is used for nonhomogeneous soils by determining that portion of the surface freezing (or thawing) index required to penetrate each layer. The sum of the thicknesses of all the frozen (or thawed) layers is the depth of freeze (or thaw). The partial freezing (or thawing) index required to penetrate the top layer is given by

$$F_1 \text{ (or } I_1) = \frac{L_1 d_1}{24\lambda_1^2}\left(\frac{R_1}{2}\right) \qquad \text{(eq 3-10)}$$

where

d_1 = thickness of first layer (ft)

$R_1 = d_1/K_1$ = thermal resistance of first layer.

The partial freezing (or thawing) index required to penetrate the second layer is

$$F_2 \text{ (or } I_2) = \frac{L_2 d_2}{24\lambda_2^2}\left(R_1 + \frac{R_2}{2}\right). \quad \text{(eq 3-11)}$$

The partial index required to penetrate the n^{th} layer is:

$$F_n \text{ (or } I_n) = \frac{L_n d_n}{24\lambda_n^2}\left(\Sigma R + \frac{R_n}{2}\right) \quad \text{(eq 3-12)}$$

where ΣR is the total thermal resistance above the n^{th} layer and equals

$$R_1 + R_2 + R_3 \dots + R_{n-1}. \quad \text{(eq 3-13)}$$

The summation of the partial indexes,

$$F_1 + F_2 + F_3 \dots + F_n (\text{or } I_1 + I_2 + I_3 \dots + I_n) \quad \text{(eq 3-14)}$$

is equal to the surface freezing index thawing index).

a. In this *example*, determine the depth of thaw penetration beneath a bituminous concrete pavement for the following conditions:

—Mean annual temperature (MAT) = 12°F.

—Air thawing index (I) = 780 degree-days.

—Average wind speed in summer = 7.5 miles per hour (mph).

—Length of thaw season (t) = 105 days.

—Soil boring log:

Since a wind speed of 7-1/2 mph results in an n-factor of 2.0 (fig. 2-10), a surface thawing index nI of 1560 degree-days is used in the computations. The v_s, v_o and α values are determined in the same way as those for the homogeneous case:

$$v_s = 1560/105 = 14.8°F \quad \text{(eq 3-15)}$$

$$v_o = 12.0 - 32.0 = 20.0°F \quad \text{(eq 3-16)}$$

$$\alpha = 20.0/14.8 = 1.35. \quad \text{(eq 3-17)}$$

The thermal properties C, K and L of the respective layers are obtained from figures 2-1 through 2-8.

b. Table 3-1 facilitates solution of the multilayer problem, and in the following discussion, layer 3 is used to illustrate quantitative values. Columns 9, 10, 12 and 13 are self-explanatory. Column 11, $\overline{L}$, represents the average value of L for a layer and is equal to $\Sigma Ld/\Sigma d$ (2581/5.0 = 517). Column 14, $\overline{C}$, represents the average value of C and is obtained from $\Sigma Cd/\Sigma d$ (145/5.0 = 29). Thus $\overline{L}$ and $\overline{C}$ represent weighted values to a depth of thaw penetration given by Σd, which is the sum of all layer thicknesses to that depth.

The fusion parameter μ for each layer is determined from

$$v_s(\overline{C}/\overline{L}) = 14.8\ (29/517) = 0.83. \quad \text{(eq 3-18)}$$

The λ coefficient is equal to 0.508 from figure 3-1. Column 18, R_n, is the ratio d/K and for layer 3 equals (3.0/2.0) or 1.5. Column 19, ΣR, represents the sum of the R_n values above the layer under consideration. Column 20, $\Sigma R + (R_n/2)$, equals the sum of the R_n values above the layer plus one-half the R_n value of the layer being considered. For layer 3 this is [1.32 + (1.50/2)] = 2.07. Column 21, nI, represents the number of degree-days required to thaw the layer being considered and is determined from

Layer	Depth (ft)	Material*	Dry unit weight (lb/ft³)	Water content (%)
1	0.0-0.4	Asphaltic concrete	138	--
2	0.4-2.0	GW-GP	156	2.1
3	2.0-5.0	GW-GP	151	2.8
4	5.0-6.0	SM	130	6.5
5	6.0-8.0	SM-SC	122	4.6
6	8.0-9.0	SM	116	5.2

*In accordance with Unified Soil Classification System.

Table 3-1. Multilayer solution of modified Berggren equation.

Table 3-1. Multilayer solution of modified Berggren equation (U.S. Army Corps of Engineers).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Layer	γ_d	w	d	Σd	C	K	L	Ld	ΣLd	$\bar{L}$	Cd	ΣCd	$\bar{C}$	μ	λ	λ^2	R_n	ΣR	$\Sigma R + \frac{R_n}{2}$	nI	ΣnI
1	138	--	0.4	0.4	28	0.86	0	0	0	--	12	12	--	--	--	--	0.46	0	0.23	--	--
2	156	2.1	1.6	2.0	29	1.85	470	751	751	376	46	58	29	1.15	0.455	0.207	0.86	0.46	0.89	134	134
3	151	2.8	3.0	5.0	29	2.00	610	1830	2581	517	87	145	29	0.83	0.508	0.258	1.50	1.32	2.07	612	746
4	130	6.5	1.0	6.0	28	1.65	1220	1220	3801	633	28	173	29	0.68	0.537	0.288	0.60	2.82	3.12	551	1297
5a	122	4.6	1.0	7.0	25	0.64	808	808	4609	658	25	198	28	0.63	0.552	0.305	1.56	3.42	4.20	465	1762
5b	122	4.6	0.6	6.6	25	0.64	808	485	4286	650	15	188	28	0.64	0.550	0.303	0.94	3.42	3.89	260	1557

$\alpha = 1.35$ $v_s = 14.8°F$ $nI = 1560$ degree-days

$$I_2 = \frac{(470)(1.6)}{(24)(0.207)}(0.89) = 134$$

$$I_3 = \frac{(610)(3.0)}{(24)(0.258)}(2.07) = 612$$

$$I_4 = \frac{(1220)(1.0)}{(24)(0.288)}(3.12) = 551$$

$$I_{5a} = \frac{(808)(1.0)}{(24)(0.305)}(4.20) = 465$$

$$I_{5b} = \frac{(808)(0.6)}{(24)(0.303)}(3.89) = 260$$

<u>Total thaw penetration = 6.6 feet</u>

$$nI = \frac{Ld}{24\lambda^2}\left(\Sigma R + \frac{R_n}{2}\right). \qquad \text{(eq 3-19)}$$

For layer 3,

$$nI_3 = \frac{(610)(3.0)}{24(0.508)^2}(2.07) \qquad \text{(eq 3-20)}$$

$$= 612 \text{ degree-days}$$

The summation of the number of degree-days required to thaw layers 1 through 4 is 1297, leaving (1560 - 1297 =) 263 degree-days to thaw a portion of layer 5. A trial-and-error method is used to determine the thickness of the thawed part of layer 5. First, it is assumed that 1.0 feet of layer 5 is thawed (designated as layer 5a). Calculations indicate 465 degree-days are needed to thaw 1.0 foot of layer 5 or (465 - 263 =) 202 degree-days more than available. A new layer, 5b, is then selected by the following proportion

$$(263/465)1.0 = 0.57 \text{ ft (try 0.6 ft)}. \qquad \text{(eq 3-21)}$$

This new thickness results in 260 degree-days required to thaw layer 5b or 3 degree-days less than available. Further trial-and-error is unwarranted and the total estimated thaw penetration would be 6.6 feet. A similar technique is used to estimate frost penetration in a multilayer soil profile.

3-5. Effect of snow and vegetative cover.
Thermal properties of snow and vegetative covers are extremely variable in both time and space. Both materials tend to act as insulators and retard heat transfer at the air-ground interface. In freeze-thaw computations, snow and vegetative surface materials are treated as separate layers in the multilayer solution of the modified Berggren equation, with snow cover thickness estimated seasonally. The tabulation below presents average thermal properties of snow applicable for calculation in the noted regions if a better data base is not available. In the absence of site-specific data, figures 2-5 and 2-6 should be used to estimate the thermal conductivities of vegetative surface cover.

3-6. Surface temperature variations.
The temperatures at the air-ground interface are subject to daily and seasonal fluctuations. Precipitation, insolation, air temperature and turbulence contribute to these variations in surface temperature. To facilitate mathematical calculations, two assumptions are commonly made regarding the temperatures at the upper boundary: 1) a sudden step change occurs in surface temperature or 2) the surface temperature change is sinusoidal. The sinusoidal variation of temperature over a year closely approximates actual conditions; however, it is amenable to hand calculations only if latent heat effects are negligible. Solutions and examples for both conditions are given below.

a. Sudden step change. This involves a sudden change in the surface temperature of a mass that was initially at a constant, uniform temperature. The sudden step change was used to establish the boundary conditions for heat flow in the modified Berggren equation given in paragraph 3-2. If the influence of latent heat is not involved, or is assumed negligible, the following equation may be used:

$$T_{(x,t)} = T_s + (T_o - T_s)\,\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right) \qquad \text{(eq 3-22)}$$

Region	Unit weight (lb/ft^3)	K (Btu/ft) hr °F)	C (Btu/ft^3 °F)	L (Btu/ft^3)
Interior Alaska	16	0.11	8	2300
Canadian Archipelago, N. Alaskan coast, and temperate regions	20	0.18	10	2880
Northern Greenland	22	0.20	11	3170

where

$T_{(x,t)}$ = temperature at depth x, at time (°F)

T_s = suddenly applied constant surface temperature (°F)

T_o = initial uniform temperature of the mass (°F)

erf = mathematical expression, termed the error function, which is frequently used in heat flow computations (dimensionless)

x = depth below surface (ft)

a = thermal diffusivity of the mass (ft^2/day = K/C)

t = time after application of sudden change in surface temperature (days).

Figure 3-2 gives the relationship between $(x/2\sqrt{at})$ and $\text{erf}(x/2\sqrt{at})$. The expression for the error function is shown in appendix A and figure 3-2. In this *example* of a sudden step change, a highly frost-susceptible subgrade is covered with a 2-foot thick, non-frost-susceptible gravel pad. Both soils are at an initial temperature of 20°F. If the

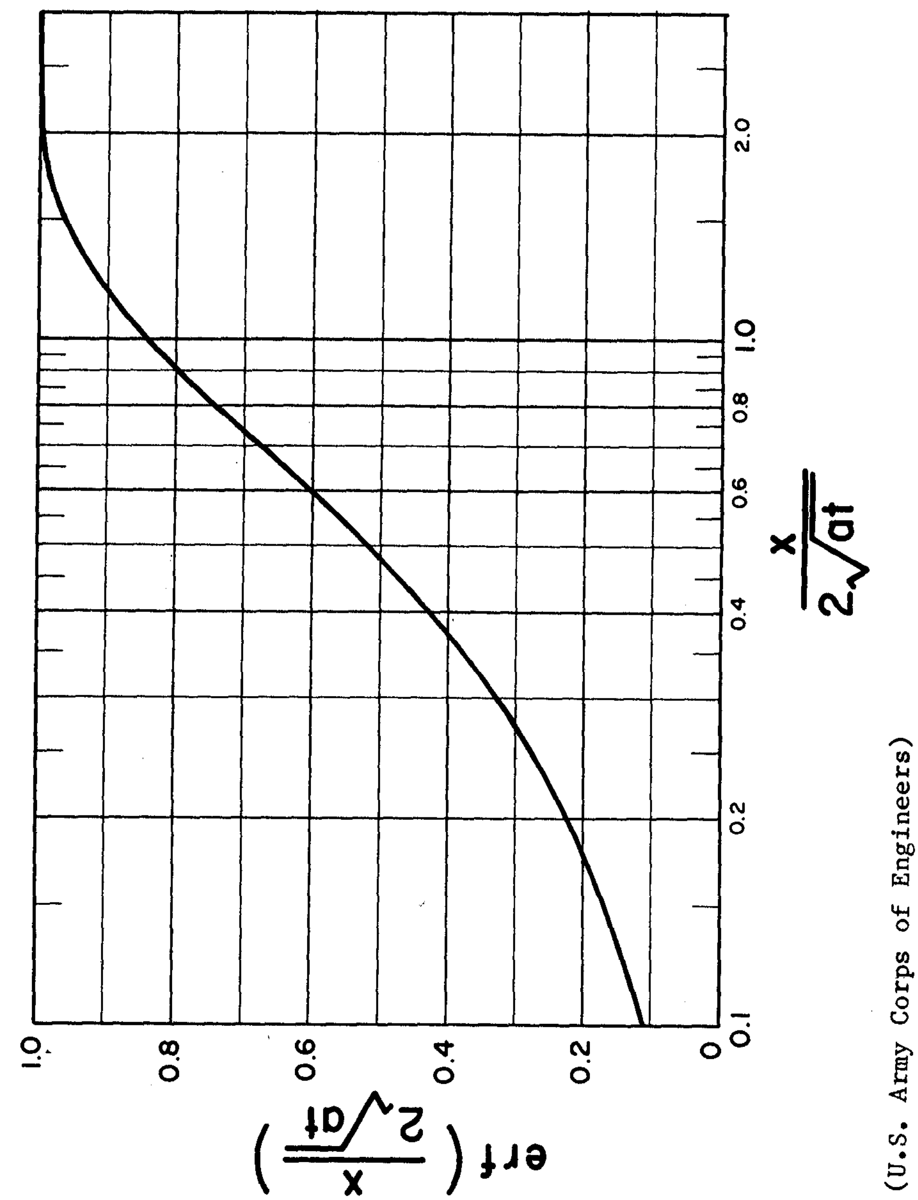


Figure 3-2. Relationship between $(x/2\sqrt{at})$ and $\text{erf}(x/2\sqrt{at})$.

surface of the gravel is suddenly heated to and maintained at 70°F for a number of days, estimate the temperature at the gravel-subgrade interface after one day. The gravel material is very dry and latent heat may be ignored. The thermal conductivity of the gravel is 1.0 Btu/ft hr °F and the volumetric heat capacity is 25 Btu/ft^3. The thermal diffusivity of the gravel is (K/C = 1.0/25) == 0.04 ft^2/hr., or 0.96 ft^2/day, and $x/2\sqrt{at} = (2.0/2\sqrt{0.96 \times 1}) = 1.02$. From figure 3-2, erf$(x/2\sqrt{at})$ is equal to 0.85, and the interface temperature T is [70 + (20 - 70)0.85] = 27.5°F.

b. Sinusoidal change. A surface temperature variation that is nearly sinusoidal repeats itself periodically for a surface exposed to the atmosphere. For most problems in this manual, the sinusoidal variation of concern occurs over an annual cycle. If latent heat is not involved or is assumed negligible, the following equation may be used:

$$A_x = A_o \exp\left(-x\sqrt{\frac{\pi}{aP}}\right) \qquad \text{(eq 3-23)}$$

where

A_x = amplitude of temperature wave at depth x (°F).

A_o = amplitude of the surface temperature wave above or below the average annual temperature (°F)

x = depth below surface (ft)

a = thermal diffusivity of the mass (ft^2/day)

P = period of sine wave (365 days).

The sinusoidal temperature pattern is assumed to exist at all levels to a depth where there is no temperature change. The temperature waves lag behind the surface wave, and the amplitude of the sinusoidal waves decreases with depth below the surface. The phase lag is determined by $t_x = (x/2)(\sqrt{365/\pi a})$. Typical temperature-time curves for a surface and at a depth x are shown in figure 3-3.

In the following *example* of a sinusoidal temperature change, the surface temperature of an 8-foot-thick concrete slab varies from 60° to -40°F during the year. Determine the maximum temperature at the base of the slab assuming a diffusivity of 1.0 ft^2/day for the concrete. The average annual temperature is [60 + (-40)]/2 = 10°F and the surface amplitude is (60 - 10) = 50°F. The amplitude at an 8-foot depth equals

$$A_x = 50 \exp\left[-8\sqrt{\frac{\pi}{(1.0)(365)}}\right] = 50\,e^{-0.742} = 24°\text{F}. \qquad \text{(eq 3-24)}$$

The maximum temperature at 8 feet is (10 + 24) = 34°F. The time lag t_x between the maximum temperature at the surface and 8 feet is

$$t_x = \frac{8}{2}\sqrt{\frac{365}{\pi(1.0)}} = 43 \text{ days (about 6 weeks).} \qquad \text{(eq 3-25)}$$

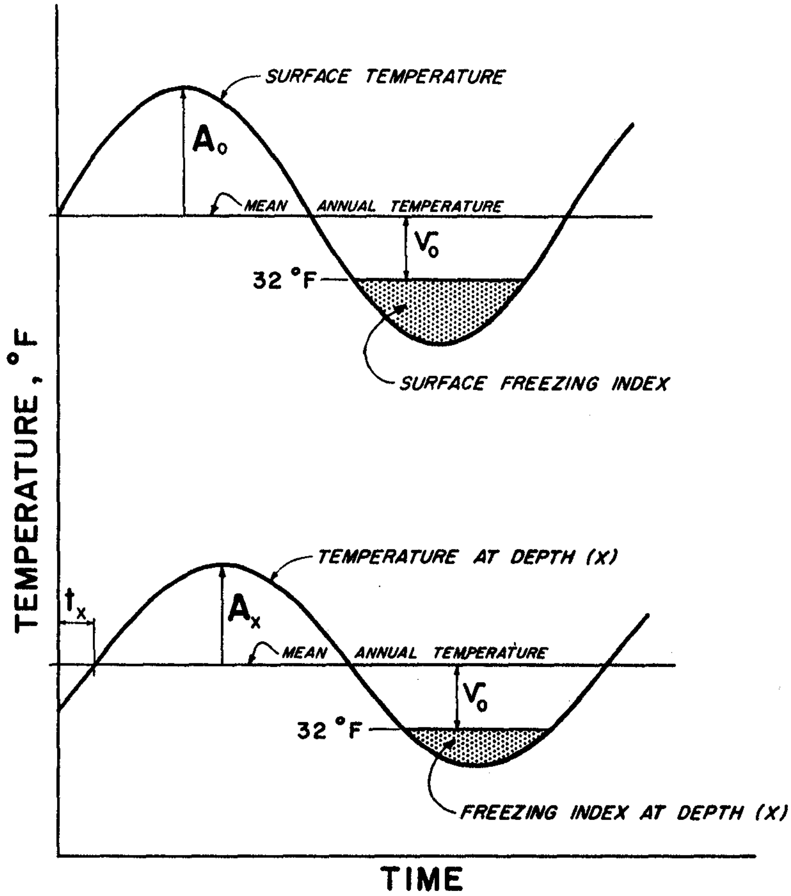
(Note: latent heat would increase the time lag and decrease the amplitude.)

c. Nonuniform layers. The method of equivalent thickness is used to find the temperature at a point below a number of layers of different thermal properties. This technique assumes a negligible effect of latent heat, and involves reduction of each layer to an equivalent material thickness by setting the ratio of the thicknesses equal to the ratio of square roots of the thermal diffusivities. For *example*, determine the equivalent gravel thickness for the three layers shown, assuming all materials are unfrozen.

The following table shows that 4.75 feet of the nonuniform materials can be considered equivalent to 5.4 feet of gravel for heat-flow purposes. This equivalent thickness and the thermal diffusivity of the gravel are used to calculate temperatures at the base of the gravel layer by either the step-change or sinusoidal method.

3-7. Converting indexes into equivalent sine wave of temperature.

Some problems may require the use of the sinusoidal temperature variation technique, given only the freezing or thawing indexes and the average annual temperature. These indexes may be converted into a sine curve of temperature to give the same index values and the sam emean temperature. For example, convert the monthly average temperature data for Fairbanks, Alaska, shown in figure 2-9 into an equivalent sine wave. The relationship between the sinusoidal amplitude, freezing index, thawing index and aver-



(U.S. Army Corps of Engineers)

Figure 3-3. Sinusoidal temperature pattern.

Material	*Dry Unit weight,* γ_d (lb/ft^3)	*Water content, w (%)*	*Thermal conductivity, K (Btu/ft hr° F)*	*Volumetric heat capacity, C (Btu/ft^3 ° F)*	*Thermal diffusivity, a = K/C* (ft^2/hr)
Concrete	--	--	1.0	33.0	0.033
Sand	120	2	0.8*	23 †	0.035
Gravel	135	4	1.5*	28 †x	0.054

*From figure 2-2.
†$C = \gamma_d\ (0.17 + w/100)$.

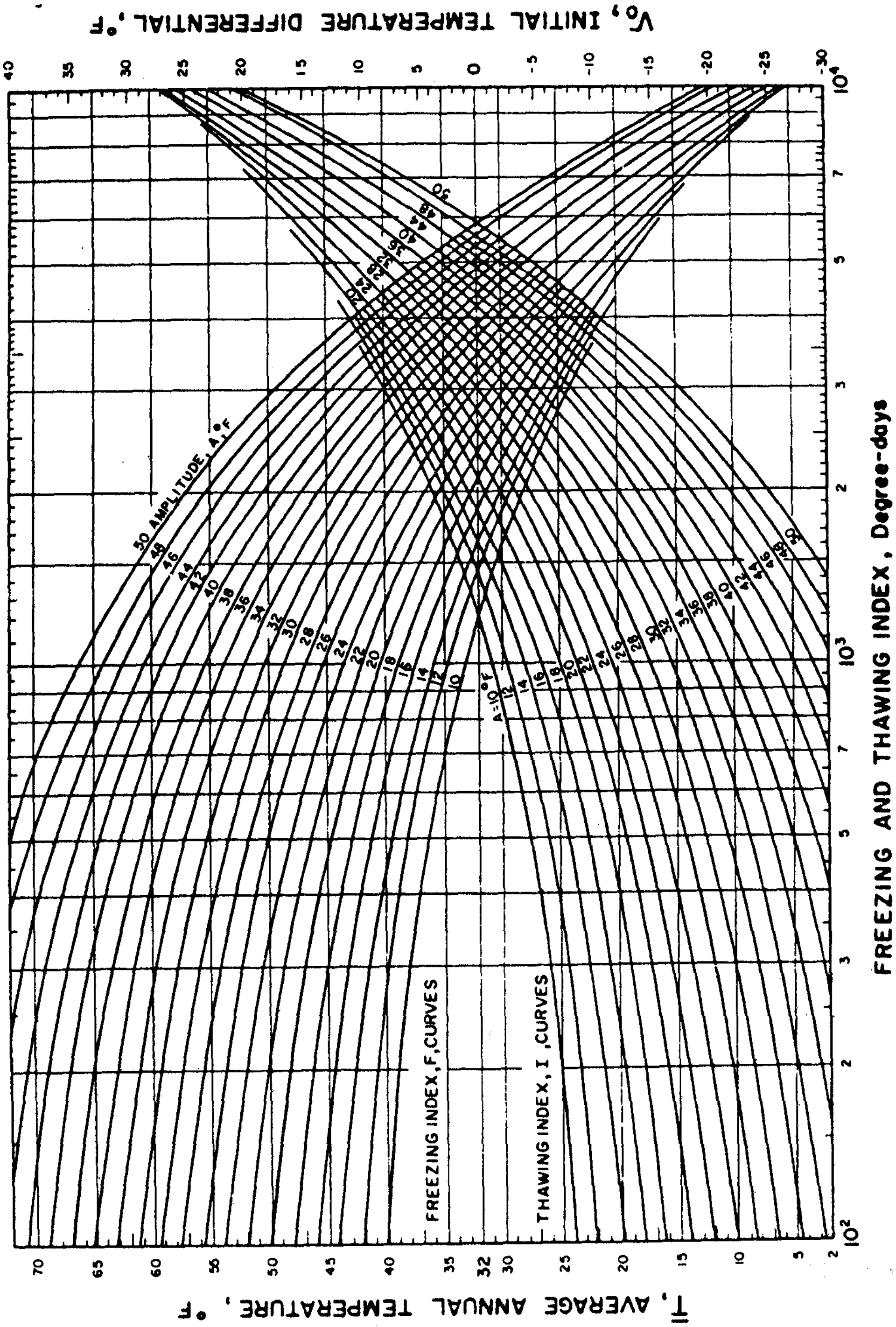
Material	*Thickness (ft)*	*Thermal diffusivity (ft^2/hr)*	$\frac{\sqrt{a_g}}{\sqrt{a_m}}$*	*Equivalent gravel thickness (ft)*
Concrete	1.75	0.033	1.3	2.3 (1.3 × 1.75)
Sand	0.50	0.035	1.2	0.6
Gravel	2.50	0.054	1.00	2.50
Total thickness	4.75			5.4

*The subscript g refers to the gravel layer and the subscript m refers to the other material layer.

age annual temperature is shown in figure 3-4.

The average temperature at Fairbanks for October 1949 to September 1950 was 27°F. By use of the freezing index of 5240 degree-days, the sinusoidal amplitude is found to be 37.0°F. The equation of the sine wave is

$$T = 27.0 - 37.0 \sin 2\pi ft \quad \text{(eq 3-26)}$$
$$= 27.0 - 37.0 \sin 0.0172\, t \text{ (radians)}$$



(U.S. Army Corps of Engineers)

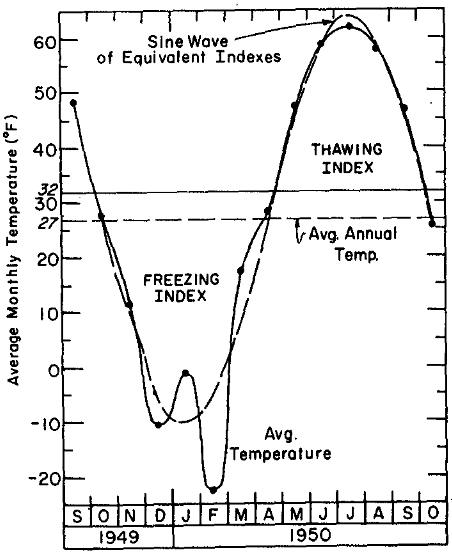
Figure 3-4. Indexes and equivalent sinusoidal temperature.

where

f = frequency, 1/365 cycles per day

t = time from origin in days. (Origin of curve is located at a point where T intersects the average annual temperature on its way downward toward the yearly minimum.)

If the thawing index of 3240 degree-days had been used, the sinusoidal temperature amplitude would be 35.5°F. The actual temperature curve for Fairbanks, Alaska, and the equivalent sine wave computed from the freezing index are plotted in figure 3-5. This illustration makes use of air indexes but a sine wave could be determined for surface indexes by multiplying the air indexes with appropriate n-factors. Note that the mean annual ground surface temperature may be substantially different (frequently higher) from the mean annual air temperature because the freezing n-factor is generally not equal to the thawing n-factor. If the long-term mean monthly temperature had been used instead of the average monthly temperatures for the 1949-1950 period, the correlation between the actual temperature curve and the equivalent sine curve would practically coincide, as shown in figure 3-6.



(U.S. Army Corps of Engineers)

Figure 3-5. Average monthly temperatures for 1949-1950 and equivalent sine wave, Fairbanks, Alaska.

3-8. Penetration of freeze or thaw beneath buildings.

The penetration of freeze or thaw beneath buildings depends largely on the presence or absence of an airspace between the building floor and the ground as discussed below and in TM 5-852-4/AFM 88-19, Chapter 4.

a. Building floor placed on ground. When the floor of a heated building is placed directly on frozen ground, the depth of thaw is determined by the same method as that used to solve a multilayer problem when the surface is exposed to the atmosphere, except that the thawing index is replaced by the product of the time and the differential between the building floor temperature and 32°F. For *example*, estimate the depth of thaw after 1 year for a building floor consisting of 8 inches of concrete, 4 inches of insulation and 6 inches of concrete, placed directly on a 5-foot-thick sand pad overlying permanently frozen silt for the following conditions:

—Mean annual temperature = 20°F.

—Building floor temperature = 65°F.

—Sand pad: γ_d = 133 lb/ft^3, w = 5%.

—Frozen silt: γ_d = 75 lb/ft^3, w = 45%.

—Concrete: K = 1.0 Btu/ft hr °F, C = 30 Btu/ft^3 °F.

—Insulation: K = 0.033 Btu/ft hr °F, C = 1.5 Btu/ft^3 °F.

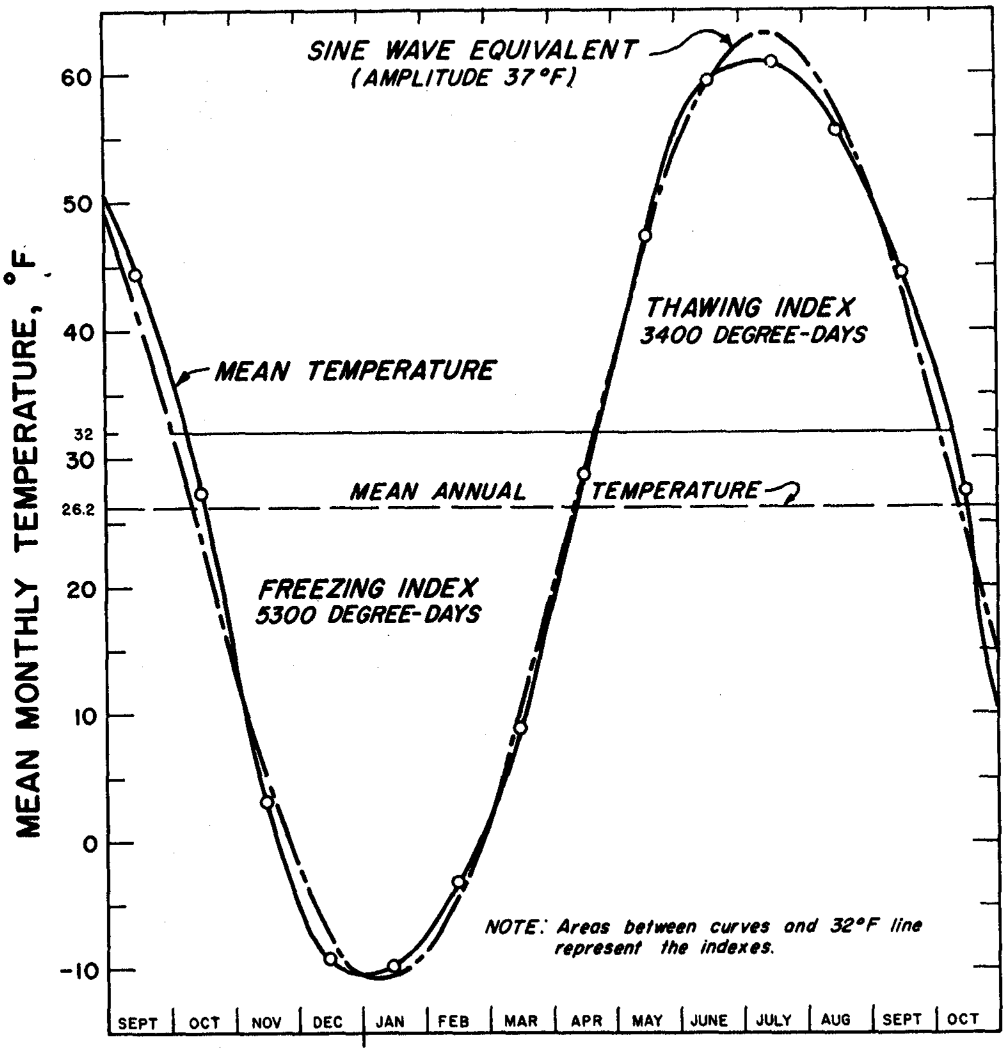
The resistances of the three floor layers are in series, and the floor resistance R_f is the sum of the three layer resistances:

$$R_f = \frac{d}{k} + \frac{8}{(12)(1.0)} + \frac{4}{(12)(0.033)} + \frac{6}{(12)(1.0)} = 11.2\ \text{ft}^2\ \text{ft}^\circ \qquad \text{(eq 3-27)}$$

The average volumetric heat capacity of the floor system is

$$C_f = \frac{(30)(8) + (1.5)(4) + (30)(6)}{8 + 4 + 6} = 23.7\ \text{Btu/ft}^3\ {}^\circ\text{F} \qquad \text{(eq 3-28)}$$

The solution to this problem, shown in table 3-2, predicts a total thaw depth of 7.8 feet. This solution did not consider edge effects, i.e., a long narrow building will have lesser depth of thaw than a square building with the same floor because of the difference in lateral heat flow.



(U.S. Army Corps of Engineers)

Figure 3-6. Long-term mean monthly temperatures and equivalent sine wave, Fairbanks, Alaska.

b. Airspace below building

(1) An unskirted airspace between the heated floor of building and the ground will help prevent degradation of underlying permafrost. The airspace insulates the building floor from the ground and acts as a convective passage for flow of cold air that dissipates heat from the floor system and the ground. The depth of thaw is calculated by means of the modified Berggren equation for either a homogeneous or multilayered soil system, as applicable. An n-factor of 1.0 is recommended to determine the surface thawing index beneath the shaded area of an elevated building.

(2) There is no simple mathematical expression for analyzing the heat flow in a ventilated floor system that has ducts or pipes installed within the floor or at some depth beneath the floor, with air circulation induced by stack effect. The depth to which freezing temperatures will penetrate is computed with the modified Berggren equation, except that the air freezing index at the outlet governs. This index is influenced by a number of design variables, i.e., average daily air temperatures, inside building temperatures, floor and duct or pipe system design, temperature and velocity of air in the system, and stack height. Cold air

Table 3-2. Thaw penetration beneath a slab-on-grade building constructed on permafrost.

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Table 3-2. Thaw penetration beneath a slab-on-grade building constructed on permafrost (U.S. Army Corps of Engineers).

Layer	γ_d	w	d	Σd	C	K	L	Ld	ΣLd	$\bar{L}$	Cd	ΣCd	$\bar{C}$	μ	λ	λ^2	R_n	ΣR	$\Sigma R + \frac{R_n}{2}$	nI	ΣnI
Floor	--	--	1.5	1.5	24	--	0	0	0	0	36	--	--	--	--	--	11.20	0	0	--	--
Sand	133	5.0	5.0	6.5	28	1.54	960	4800	4800	738	140	176	27	1.21	0.68	0.463	3.25	11.20	12.82	5540	5540
Silt a	72	45.0	1.5	8.0	37	0.90	4650	6970	11770	1470	55	231	29	0.65	0.77	0.593	1.67	14.45	15.29	7480	13020
Silt b	72	45.0	1.3	7.8	37	0.90	4650	6050	10850	1390	48	224	29	0.69	0.765	0.586	1.44	14.45	15.17	6520	12060

$v_o = 32 - 20 = 12°F$ $\quad$ $v_s = 65-32 = 33°F$ $\quad$ $\alpha = 12/33 = 0.36$

Surface thawing index (nI) = 33 x 365 = 12050 degree-days

$$nI\,(\text{Sand}) = \frac{4800}{24\,(0.463)}\,(12.82) = 5540 \text{ degree-days}$$

$$nI\,(\text{Silt a}) = \frac{6970}{24\,(0.593)}\,(15.29) = 7480 \text{ degree-days}$$

$$nI\,(\text{Silt b}) = \frac{6050}{24\,(0.586)}\,(15.17) = 6520 \text{ degree-days}$$

<u>Total thaw penetration = 7.8 feet</u>

passing through the ducts acquires heat from the duct walls and experiences a temperature rise as it moves through the duct, and the air freezing index is reduced at the outlet. Field observations indicate that the inlet air freezing index closely approximates the site air freezing index. The freezing index at the outlet must be sufficient to counteract the thawing index and ensure freeze-back of foundation soils.

(3) As an *example*, determine the required thickness of a gravel pad beneath the floor section shown in figure 3-7 to contain all thaw penetration. Also determine the required stack height to ensure freeze-back of the pad on the outlet side of the ducts. The conditions for this example follow:

- Duct length, l = 220 ft.
- Gravel pad: γ_d = 125 lb/ft^3, w = 2.5%.
- Outlet mean annual temperature = 32°F 0, w = 2.5%. (conservative assumption).
- Minimum site freezing index = 4000 degree-days.
- Freezing season = 215 days.
- Thawing season = 150 days (period during which ducts are closed).
- Building floor temperature = 60°F.
- Thermal conductivity of concrete, K_c = 1.0 Btu/ft hr °F.
- Thermal conductivity of insulation, K_i = 0.033 Btu/ft hr °°°°F.

(a) The required thickness is determined by the following equation, derived from the modified Berggren equation:

$$X = KR_f \left[\sqrt{1 + \frac{48r^2 I_f}{KL(R_f)^2}} - 1 \right] \qquad \text{(eq 3-29)}$$

Figure 3-7. Schematic of ducted foundation.

where

K = average thermal conductivity of gravel
= 1/2(0.7 + 1.0) = 0.85 Btu/ft hr °F

R_T = thermal resistance of floor system

$$= \frac{18}{12(1.0)} + \frac{4}{12(0.033)} + \frac{12}{12(1.0)}$$

$$= 12.5 \text{ ft}^2 \text{ hr °F/Btu} \qquad \text{(eq 3-30}$$

(In the computations the dead airspace is assumed equivalent to the thermal resistance of concrete of the same thickness.)

λ = factor in modified Berggren equation = 0.97 (conservative assumption)

I_f = thawing index at floor surface = (60 - 32)(150) = 4200 degree-days

L = latent heat of gravel = 144(125)(0.025) = 450 Btu/ft³

then

$$X = (0.85)(12.5)\left[\sqrt{1 + \frac{(48)(0.97)^2(4200)}{(0.85)(450)(12.5)^2}} - 1\right]$$

$$= 11.0 \text{ ft.} \qquad \text{(eq 3-31)}$$

(b) Thus the total amount of heat to be removed from the gravel pad by cold-air ventilation during the freezing season with ducts open is equal to the latent and sensible heat contained in the thawed pad. The heat content per square foot of pad is determined as follows:

- Latent heat, (X)(L) = (11.0)(450) = 4950 Btu/ft²
- Sensible heat (10 percent of latent heat, based upon experience) = 495
- Total heat content:
5445 Btu/ft².

The ducts will be open during the freezing season (215 days), and the average rate of heat flow from the gravel during this season is equal to 5445/215 × 24 = 1.0 Btu/ft² hr. The average thawing index at the surface of the pad is

$$\frac{LX^2}{48\lambda^2 K} = \frac{(450)(11.0)^2}{48(0.97)^2(0.85)} = 1420 \text{ degree-days.} \qquad \text{(eq 3-32)}$$

This thawing index must be compensated for by an equal freezing index at the duct outlet on the surface of the pad to assure freeze-back. The average pad surface temperature at the outlet end equals the ratio

$$\frac{\text{Required freezing index}}{\text{Length of freezing season}}$$

$$= \frac{1420}{215}$$

= 6.6°F below 32°F or 25.4°F.

The inlet air during the freezing season has an average temperature of

$$\frac{\text{Air freezing index}}{\text{Length of freezing season}} = \frac{4000}{215}$$

= 18.6°F below 32°F or 13.4°F.

Therefore, the average permissible temperature rise T_R along the duct is (25.4 - 13.4) = 12.0°F.

(c) The heat flowing from the floor surface to the duct air during the winter is equal to the temperature difference between the floor and duct air divided by the thermal resistance between them. The thermal resistance R is calculated as follows:

$$R = \frac{X_c}{K_c} + \frac{X_i}{K_i} + \frac{1}{h_{rc}} = \frac{14}{(12)(1.0)}$$

$$+ \frac{4}{(12)(0.033)} + \frac{1}{1.0} = 12.3 \text{ hr ft}^2 \text{ °F/Btu} \qquad \text{(eq 3-33)}$$

where

X_c = thickness of concrete (ft)

X_i = thickness of insulation (ft)

h_{rc} = surface transfer coefficient between duct wall and duct air

(For practical design, h_{rc} = 1.0 Btu/ft² hr °F and represents the combined effect of convection and radiation. At much higher air velocities, this value will be slightly larger; however, using a value of 1.0 will lead to conservative designs). The average heat flow between the floor and inlet duct air is [(60 - 13.4)/12.3] = 3.8 Btu/ft² hr, and between the floor and outlet duct air is [(60 - 25.4)/12.3] = 2.8 Btu/ft² hr. Thus the average rate of heat flow from the gravel pad to the duct air is 1.0 Btu/ft² hr. The total heat flow ϕ to the duct air from the floor and gravel pad is (3.3 + 1.0) = 4.3 Btu/ft² hr. The heat flow to the duct air must equal the heat removed by the duct air·

Heat added = heat removed

$$\phi \ell m = 60V A_d \rho c_p T_R. \qquad \text{(eq 3-34)}$$

Thus the average duct air velocity required to extract this quantity of heat (4.3 Btu/ft² hr) is determined by the equation:

$$V = \frac{\phi \ell m}{60\, A_d\, \rho\, C_p T_R} \text{ ft/minute} \qquad \text{(eq 3-35)}$$

where

ϕ = total heat flow to duct air (4.3 Btu/ft² hr)

ℓ = length of duct (220 ft)

m = duct spacing (2.66 ft)

A_d = cross-sectional area of duct (1.58 ft²)

ϕ = density of air (0.083 lb/ft³ [figure 3-10])

c_p = specific heat of air at constant pressure (0.24 Btu/lb °F)

T_R = temperature rise in duct air (12°F).

Substitution of appropriate values gives a required air velocity

$$V = \frac{(4.3)(220)(2.66)}{(60)(1.58)(0.083)(0.24)(12.0)}$$

$$= 111 \text{ ft/minute.} \qquad \text{(eq 3-36)}$$

(d) The required air flow is obtained by a stack or chimney effect, which is related to the stack height. The stack height is determined by the equation

$$h_d = h_v + h_f \qquad \text{(eq 3-37)}$$

where

$$h_d = \frac{\rho\, \varepsilon\, H(T_c - T_o)}{5.2(T_c + 460)} \text{ inches of water}$$

(natural draft head)

ρ = density of air at average duct temperature (lb/ft³)

ε = efficiency of stack system (%). This factor provides for

Figure 3-8. Properties of dry air at atmospheric pressure.

friction losses within the chimney

H = stack height (ft)

T_c = temperature of air in stack (°F)

T_o = temperature of air surrounding stack (°F)

$$h_v = \left(\frac{V}{4000}\right)^2 \quad \text{inches of water (velocity head)}$$

V = velocity of duct air (ft/minute)

$$h_f = f' \frac{\ell_e}{D_e} h_v \quad \text{inches of water (friction head)}$$

f' = friction factor (dimensionless)

ℓ_e = equivalent duct length (ft)

D_e = equivalent duct diameter (ft).

The technique used to calculate the friction head is

$$D_e = \frac{4(\text{cross-sectional area of duct in ft}^2)}{\text{perimeter of duct in ft}} = \frac{4(1.58)}{\frac{2}{12}\left(\frac{18+20}{2}+12\right)}$$

$$= 1.22 \text{ ft.} \qquad \text{(eq 3-38)}$$

The equivalent length of the duct is equal to the actual length ℓ_s plus an allowance ℓ_b for bends and entry and exit. Each right-angle bend has the effect of adding approximately 65 diameters to the length of the duct, and entry and exit effects add about 10 diameters for each entry or exit. In this example the total allowance ℓ_b for these effects is [2(65 + 10) =] 150 diameters, which is added to the length of the straight duct. The estimated length of straight duct ℓ_s is

5 ft	(assumed inlet open length)
220 ft	(length of duct beneath floor)
15 ft	(assumed stack height)
240 ft	

$$\ell_e = \ell_s + \ell_b \qquad \text{(eq 3-39)}$$

$$\ell_e = 240 + (150 \times 1.22) = 423 \text{ ft.}$$

The friction factor f' is a function of Reynolds number N_R and the ratio e/D_e. A reasonable absolute roughness factor e of the concrete duct surface is 0.001 feet, based on field observations. Suggested values of e for other types of surfaces are given in the *ASHRAE Data and Guide Book*. The effect of minor variations in e on the friction factor is small, as noted by examining the equation below used to calculate the friction factor f'. Reynolds number is obtained from the equation

$$N_R = \frac{V(a' + 0.25 D_e)}{\nu} \qquad \text{(eq 3-40)}$$

where

$$N_R = \frac{(111 \times 60)(1.0 + 0.25 \times 1.22)}{0.49} = 17{,}700$$

V = average duct velocity (ft/hr)

a' = shortest dimension (ft)

ν = kinematic viscosity (ft^2/hr at 19.4°F [fig. 3-8]).

The friction factor f' is obtained by solving the equation

$$f' = 0.0055[1 + (20{,}000 \times \frac{e}{D_e} + \frac{10^6}{N_R})^{1/3}]$$

$$= 0.0055[1 + (20{,}000 \times \frac{0.001}{1.22} + \frac{10^6}{17{,}700})^{1/3}]$$

$$= 0.0285. \qquad \text{(eq 3-41)}$$

Therefore, the friction head is

$$h_f = f' \times \frac{\ell_e}{D_e} \times h_v \qquad \text{(eq 3-42)}$$

$$= 0.0285 \times \frac{423}{1.22} \times h_v = 9.8\, h_v.$$

The draft head required to provide the desired velocity head and to overcome the friction head is furnished by the chimney or stack effect. The draft head h_d is obtained as follows:

$$h_d = h_v + h_f = h_v + 9.8\, h_v \qquad \text{(eq 3-43)}$$

$$= 10.8\, h_v$$

$$= 10.8 \left(\frac{V}{4000}\right)^2$$

$$= 10.8 \left(\frac{111}{4000}\right)^2 = 8.31\times10^{-3} \text{ inches of water.}$$

The stack height required to produce this draft head is

$$H = \frac{5.2\, h_d(T_c + 460)}{\rho\, \varepsilon\, (T_c - T_o)}$$

$$= \frac{(5.2)(8.31\times10^{-3})(25.4 + 460)}{(0.083)(0.80)(25.4 - 13.4)}$$

$$= 26 \text{ ft} \qquad \text{(eq 3-44)}$$

where

ρ = 0.083 lb/ft^3

T_c = 25.4°F

T_o = 13.4°F

ε = 80% (found to be a reasonable design value based on observations over an entire season)

h_d = 8.31×10^{-3} inches of water.

(e) If the stack is too high for the structure, a greater thickness of insulation could be used. In this example, the effect of increasing the insulation thickness by one-half would result in lowering the stack height by five-eighths.

(f) This first approximated stack height is next incorporated in the calculation of the length of straight duct ℓ_s, and the newly obtained ℓ_e is used to recalculate the friction head h_f. By trial-and-error, the final calculated stack height is found to be 26.5 ft.

(g) The stack height is an important variable because an increase in stack height will increase the duct airflow. Circulation of air through the ducts results from 1) a density difference between the air inside the duct and that outside the building, 2) a pressure reduction at the outlet end attributable to the stack effect, 3) a positive pressure head at the inlet end when wind blows directly into the intake stack opening, and 4) a negative pressure head at the outlet when wind passes over the exhaust stack opening. Draft caused by wind is highly erratic and unpredictable and should not be considered in design; however, the vents should be cowled to take advantage of any available velocity head provided by the wind. If sufficient air cannot be drawn through ducts by natural draft, mechanical blowers could be specified or consideration given to alternating airflow in the ducts.

3-9. Use of thermal insulating materials.

An insulating layer may be used in conjunction with a non-frost-susceptible material to reduce the thickness of fill required to keep freezing or thawing temperatures from penetrating into an underlying frost-susceptible soil. As in the example of paragraph 3-8a, the thermal resistance of the pavement and insulation layers are added to obtain total resistance, and the latent heat effect of a combined pavement and insulation layer is assumed negligible. (TM 5-818-2/AFM 88-6, Chap. 4 discusses in detail the design of insulated pavements.) If the insulating material will absorb water, its insulating effectiveness will be reduced considerably (as discussed in TM 5-852-4/AFM 88-19, Chap. 4). Limited field tests indicate that the heat-flow resistance of a portland-cement-concrete pavement overlying a high-quality insulating layer is more complicated than simple addition of resistances, but until sufficient data are obtained for validation, treatment of resistances in series is recommended.

a. Example. A pavement consists of 14 inches of portland-cement concrete placed on a 6-foot gravel base course. Frost penetrated 3.2 feet into the underlying silt subgrade. Determine the thickness of insulation required to prevent frost penetration into the subgrade for the following conditions.

— Mean annual temperature = 35.3°F.
— Air freezing index = 3670 degree-days
— Freezing season = 170 days
— Concrete: K = 1.0 Btu/ft hr °F, C = 0.30 Btu/ft^3 °F
— Insulation: K = 0.024 Btu/ft hr °F, C = 0.28 Btu/ft^3 °F
— Gravel base: γ_d = 130 lb/ft^3, w = 4%
— Silt subgrade: γ_d = 100 lb/ft^3, w = 10%.

Surface freezing index = 0.75 × 3670 = 2752 degree-days. From the known data

$$v_s = \frac{2752}{170} = 16.2°F \qquad \text{(eq 3-45)}$$

$$v_o = 35.3 - 32.0 = 3.3°F \qquad \text{(eq 3-46)}$$

$$\alpha = \frac{3.3}{16.2} = 0.20 \qquad \text{(eq 3-47)}$$

b. Trial 1. Use a 2-inch layer of insulation and a 6-inch concrete leveling course.

—Pavement section: 14 inches of concrete
2 inches of insulation
6 inches of concrete leveling course

22 inches total

$$R_p = \frac{d}{K} = \frac{14}{12 \times 1.0} + \frac{2}{12 \times 0.024} + \frac{6}{12 \times 1.0}$$

$$= 8.60 \text{ hr ft}^2 \text{ °F/Btu} \qquad \text{(eq 3-48)}$$

$$C_p = \frac{(14 \times 30) + (2 \times 0.28) + (6 \times 30)}{22}$$

$$= 27.3 \text{ Btu/ft}^3 \text{ °F} \qquad \text{(eq 3-49)}$$

The calculation appears in table 3-3 and indicates that this pavement section has an excess of (2752 - 3033 =) 481 degree-days to prevent frost penetration into the silt subgrade.

c. Trial 2. Use a 1.5-inch layer of insulation and a 6-inch concrete leveling course.

—Pavement section:
14 inches of concrete
1.5 inches of insulation
6 inches of concrete leveling course

21.5 inches total

$$R_p = \frac{d}{K} = \frac{14}{12 \times 1.0} + \frac{1.5}{12 \times 0.024} + \frac{6}{12 \times 1.0}$$

$$= 6.86 \text{ hr ft}^2 \text{ °F/Btu} \qquad \text{(eq 3-50)}$$

$$C_p = \frac{(14 \times 30) + (1.5 \times 0.28) + (6 \times 30)}{21.5}$$

$$= 27.9 \text{ Btu/ft}^3 \text{ °F} \qquad \text{(eq 3-51)}$$

The calculation (see table 3-4) indicates that this pavement section will not prevent frost penetration into the silt subgrade as (2752 - 2553 =) 199 degree-days remain for subgrade penetration. The 2-inch thickness of insulation is therefore required.

Table 3-3. Insulated pavement design, no frost penetration.

Table 3-4. Insulated pavement design, frost penetration.